Useful Trigonometric Identities

Euler's Relations

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad e^{-j\theta} = \cos\theta - j\sin\theta$$

Product Identities

$$\cos(a)\sin(b) = \frac{e^{ja}+e^{-ja}}{2}\frac{e^{jb}-e^{-jb}}{2j} = \frac{\sin(a+b)-\sin(a-b)}{2}$$

$$\sin(a)\cos(b) = \frac{e^{ja}-e^{-ja}}{2j}\frac{e^{jb}+e^{-jb}}{2} = \frac{\sin(a+b)+\sin(a-b)}{2}$$

$$\sin(a)\sin(b) = \frac{e^{ja}-e^{-ja}}{2j}\frac{e^{jb}-e^{-jb}}{2j} = \frac{\cos(a-b)-\cos(a+b)}{2}$$

$$\cos(a)\cos(b) = \frac{e^{ja}+e^{-ja}}{2}\frac{e^{jb}+e^{-jb}}{2} = \frac{\cos(a+b)+\cos(a-b)}{2}$$

Sum Identities

$$\sin(a+b) = \sin(a)\cos(b)+\cos(a)\sin(b)$$

$$\sin(a-b) = \sin(a)\cos(b)-\cos(a)\sin(b)$$

$$\cos(a+b) = \cos(a)\cos(b)-\sin(a)\sin(b)$$

$$\cos(a-b) = \cos(a)\cos(b)+\sin(a)\sin(b)$$

My favorite identity

Consider the quantity: $A\cos(\theta)+B\sin(\theta)$

Now, let $A = R\sin(\phi)$ and $B = R\cos(\phi)$

Or, $R = \sqrt{A^2+B^2}$ and $\phi = \tan^{-1}\left(\frac{A}{B}\right)$

Plugging in for A and B and using the product identities:

$$\begin{aligned} A\cos(\theta)+B\sin(\theta) &= R\cos(\theta)\sin(\phi)+R\sin(\theta)\cos(\phi) \\ &= R\frac{\sin(\theta+\phi)-\sin(\theta-\phi)}{2}+R\frac{\sin(\theta+\phi)+\sin(\theta-\phi)}{2} \\ &= R\sin(\theta+\phi) \end{aligned}$$

Rewriting this:

$$\boxed{A\cos(\theta)+B\sin(\theta) = R\sin(\theta+\phi)\ ;\ R = \sqrt{A^2+B^2}\ ;\ \phi = \tan^{-1}\left(\frac{A}{B}\right)}$$