Planetary Geartrain Analysis

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Introduction

This analysis takes an in-depth look at the governing kinematics and dynamics pertaining to a planetary geartrain. Figure 1 below shows a simplified diagram of a planetary geartrain.

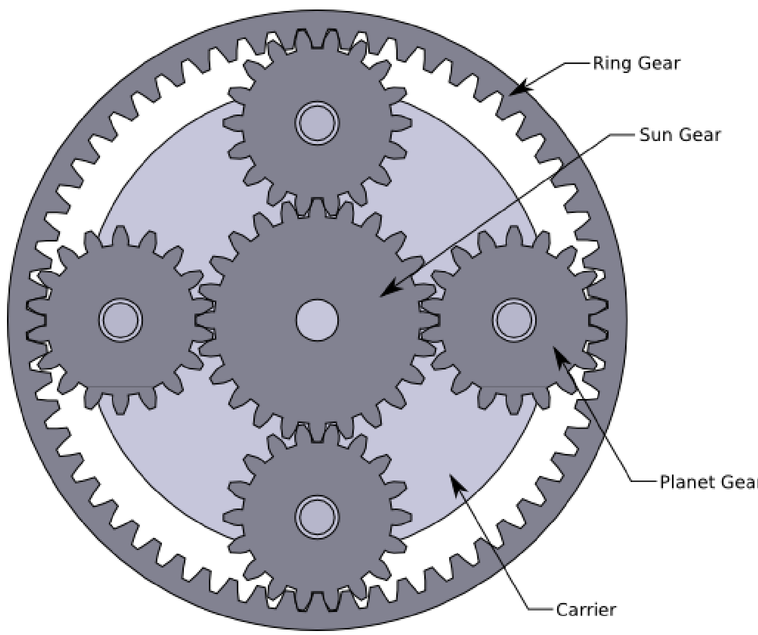

Figure 1: Simplified diagram of a planetary geartrain

Planetary kinematics

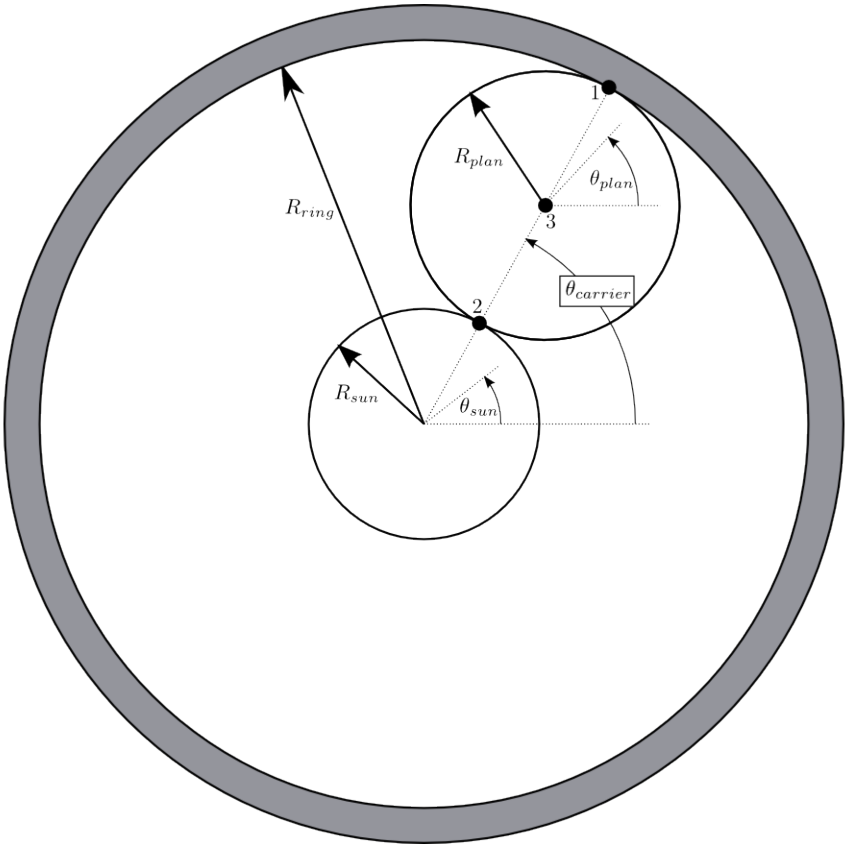

Figure 2.: Defining dimensions

- Note that point 1 on the planet gear is stationary (zero velocity) due to no-slip assumption
- Note that point 2 on the planet gear has the same velocity as point 2 on the sun gear (also due to no-slip assumption), which is given by:
  $v_{2s} = v_{2p} = R_s\dot{\theta}_s$
- The speed of point 3 on the planet gear is ½ of the speed at point 2:
  $v_{3p} = \frac{1}{2}R_s\dot{\theta}_s$
- The angular speed of the planet carrier can be found since we know the (translational) speed of the planet gear (V_{3p}):
  $$(R_s + R_p)\,\dot{\theta}_c = \frac{1}{2}R_s\dot{\theta}_s \Rightarrow \dot{\theta}_c = \frac{R_s}{2(R_s+R_p)}\dot{\theta}_s \qquad (1)$$
- The angular speed of the planet gear can be found since we know the instantaneous speeds of two points on it (namely at points 1 and 2):
  $$v_{2p} = -2R_p\dot{\theta}_p \Rightarrow \dot{\theta}_p = -\frac{R_s}{2R_p}\dot{\theta}_s \qquad (2)$$

Dynamics Analysis

Sun Gear

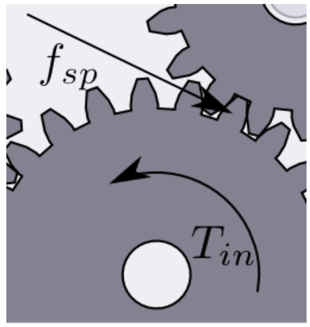

Figure 3: Sun gear free body diagram

Applying Newton's Second Law in Rotation (NSLR) to the sun gear, we have:

$$T_{in} - R_s N_p f_{sp} = J_s \ddot{\theta}_s \quad (3)$$

where T_{in} denotes the input torque (from a motor for example), N_p denotes the number of planet gears f_{sp} denotes the tangential force at the interface between the sun gear and each planet gear (we assume here that all planet gears share the load evenly), and J_s denotes the polar moment of inertia of the sun gear.

Planet Gear

For each planet gear, we can write NSLR as follows:

$$R_p f_{pr} - R_p f_{sp} = J_p \ddot{\theta}_p \quad (4)$$

where f_{pr} denotes the tangential force between each planet gear and the fixed ring gear and J_p denotes the polar moment of inertia of each planet gear.

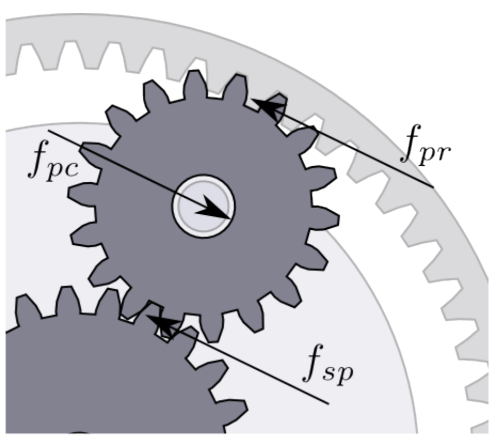

Figure 4: Planet gear free body diagram

We can also write Newton's Second Law (in translational form) in the direction of the three forces shown above (aligning the y-axis with that direction):

$$f_{sp} + f_{pr} - f_{pc} = m_p \ddot{y}_p$$

The linear acceleration, $\ddot{y}_p$, can be replaced by $(R_s + R_p)\,\ddot{\theta}_c$ to yield:

$$f_{sp} + f_{pr} - f_{pc} = m_p (R_s + R_p)\,\ddot{\theta}_c \qquad (5)$$

Planet Carrier

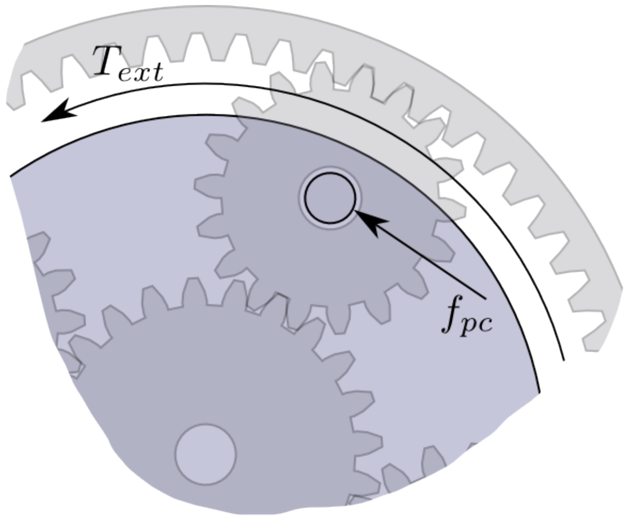

Figure 5: Planet carrier free body diagram

Applying NSLR to the planet carrier, we get:

$$T_{ext} + (R_s + R_p)\, N_p f_{pc} = J_c \ddot{\theta}_p \qquad (6)$$

Combining them all together

We now have 6 governing equations with 6 unknowns (f_{sp}, f_{pr}, f_{pc}, $\ddot{\theta}_p$, $\ddot{\theta}_c$, and $\ddot{\theta}_s$). Through successive substitution, we can express the governing dynamical equation as follows:

$$T_{in} + \frac{R_s}{2(R_s+R_p)} T_{ext} = \left[J_s + \left(\frac{R_s}{2R_p}\right)^2 N_p J_p + \frac{R_s^2}{4} N_p m_p + \left(\frac{R_s}{2(R_s+R_p)}\right)^2 J_c \right] \ddot{\theta}_s$$

Rearranging slightly, we can identify the contributions of the sun gear, planet gears, and planet carrier to the total effective inertia:

$$T_{in} + \frac{R_s}{2(R_s+R_p)} T_{ext} = \underbrace{\left[\underbrace{J_s}_{\text{sun gear}} + \underbrace{N_p \frac{R_s^2}{4} \left(\frac{1}{R_p^2} J_p + m_p \right)}_{\text{planet gears}} + \underbrace{\left(\frac{R_s}{2\,(R_s + R_p)} \right)^2 J_c}_{\text{planet carrier}} \right]}_{\text{total effective inertia}} \ddot{\theta}_s$$