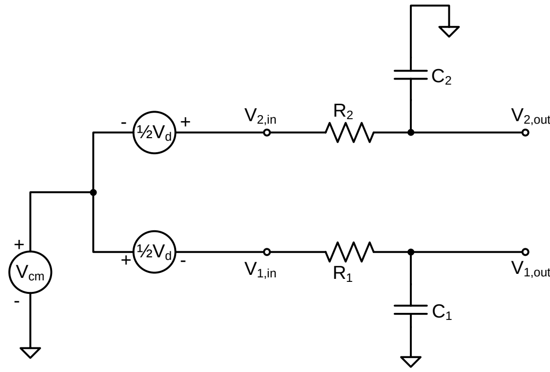
<u>Case 1:</u> RC filter on high and low-side



$$\frac{V_{2,out}}{V_{2,in}} = G_2(s) = \frac{1}{R_2C_2s+1} \qquad \frac{V_{1,out}}{V_{1,in}} = G_1(s) = \frac{1}{R_1C_1s+1}$$

The outputs are defined as follows: $V_{diff,out} \equiv V_{2,out} - V_{1,out}$ $\qquad V_{cm,out} \equiv \frac{V_{2,out}+V_{1,out}}{2} = \text{avg}(V_{2,out}, V_{1,out})$

The relevant TF's are:

$$\frac{V_{diff,out}}{V_{diff,in}} = \frac{G_2+G_1}{2} \qquad \frac{V_{cm,out}}{V_{diff,in}} = \frac{G_2-G_1}{4}$$

$$\frac{V_{diff,out}}{V_{cm,in}} = G_2 - G_1 \qquad \frac{V_{cm,out}}{V_{cm,in}} = \frac{G_2+G_1}{2}$$

Let's define: $\tau_1 \equiv R_1C_1 \qquad \tau_2 \equiv R_2C_2$

$$\frac{V_{diff,out}}{V_{diff,in}} = \frac{G_2+G_1}{2} = \frac{1}{2}\left(\frac{1}{\tau_2s+1}+\frac{1}{\tau_1s+1}\right) = \frac{1}{2}\frac{(\tau_1s+1)+(\tau_2s+1)}{(\tau_2s+1)(\tau_1s+1)} = \frac{\frac{1}{2}(\tau_1+\tau_2)s+1}{(\tau_2s+1)(\tau_1s+1)}$$

And now define: $\hat{\tau} \equiv \frac{\tau_1+\tau_2}{2} = \text{avg}(\tau_1,\tau_2)$

This yields:

$$\boxed{\frac{V_{diff,out}}{V_{diff,in}} = \frac{\hat{\tau}s+1}{(\tau_2s+1)(\tau_1s+1)}}$$

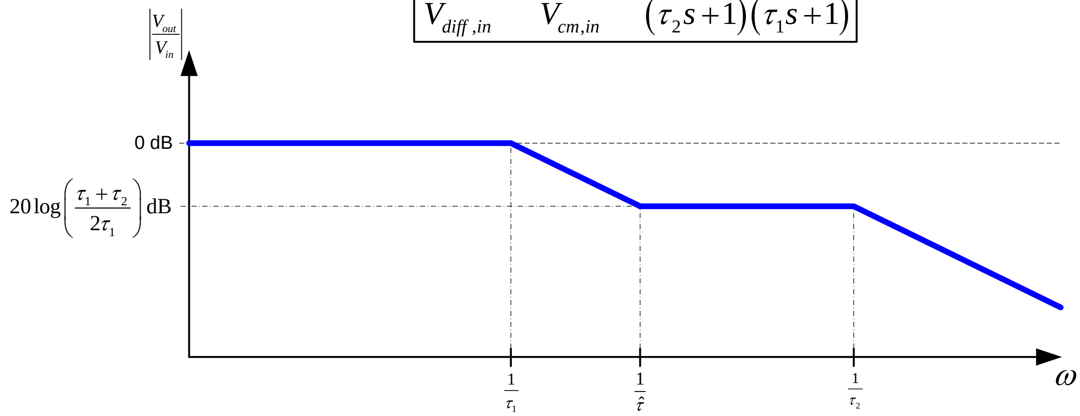
Note that the transfer function from CM input to CM output is the same.

$$\boxed{\frac{V_{diff,out}}{V_{cm,in}} = G_2 - G_1 = \frac{1}{\tau_2s+1} - \frac{1}{\tau_1s+1} = \frac{(\tau_1s+1)-(\tau_2s+1)}{(\tau_2s+1)(\tau_1s+1)} = (\tau_1-\tau_2)\frac{s}{(\tau_2s+1)(\tau_1s+1)}}$$

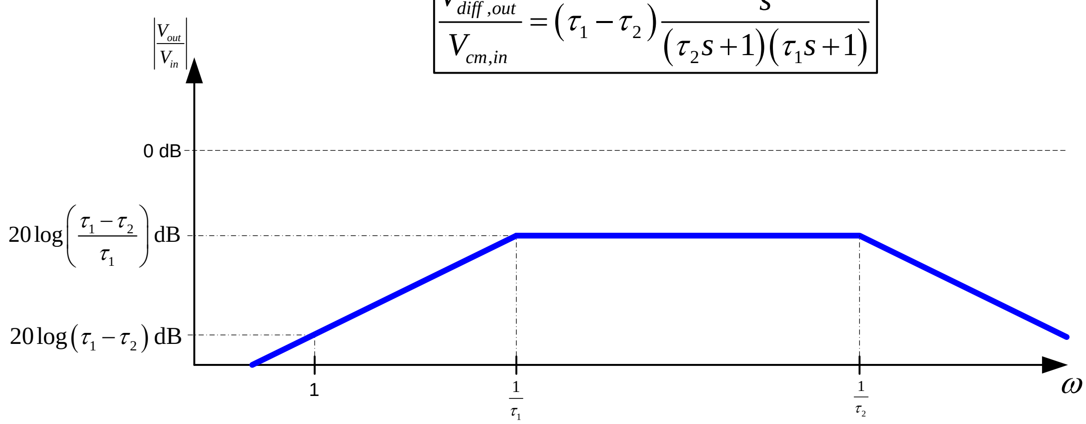
$$\boxed{\frac{V_{cm,out}}{V_{diff,in}} = \frac{G_2-G_1}{4} = \left(\frac{\tau_1-\tau_2}{4}\right)\frac{s}{(\tau_2s+1)(\tau_1s+1)}}$$

Let's look at the qualitative Bode Plot's.

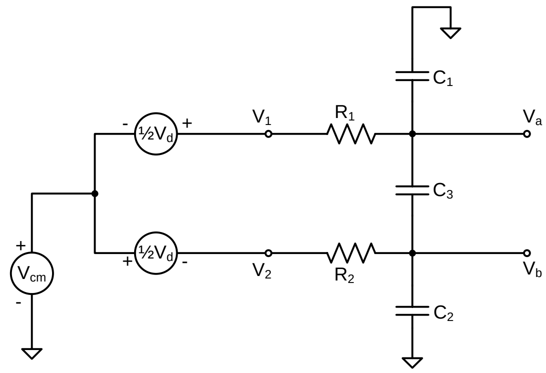
$$\frac{V_{diff,out}}{V_{diff,in}} = \frac{V_{cm,out}}{V_{cm,in}} = \frac{\hat{\tau}s+1}{(\tau_2 s+1)(\tau_1 s+1)}$$



$$\frac{V_{diff,out}}{V_{cm,in}} = (\tau_1-\tau_2)\frac{s}{(\tau_2 s+1)(\tau_1 s+1)}$$



<u>Case 2:</u> Case 1 plus C_3 across output



$$\frac{V_1 - V_a}{R_1} = C_1 V_a' + C_3 \left(V_a' - V_b' \right)$$

$$\frac{V_2 - V_b}{R_2} = C_2 V_b' + C_3 \left(V_b' - V_a' \right)$$

$$V_1 = \left[R_1 \left(C_1 + C_3 \right) s + 1 \right] V_a - R_1 C_3 s V_b$$

$$V_2 = \left[R_2 \left(C_2 + C_3 \right) s + 1 \right] V_b - R_2 C_3 s V_a$$

$$\begin{bmatrix} R_1 \left(C_1 + C_3 \right) s + 1 & -R_1 C_3 s \\ -R_2 C_3 s & R_2 \left(C_2 + C_3 \right) s + 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

In state-space form, it is...

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$

$$A = \begin{bmatrix} -\frac{C_2 + C_3}{R_1 C_{eq}} & -\frac{C_3}{R_2 C_{eq}} \\ -\frac{C_3}{R_1 C_{eq}} & -\frac{C_1 + C_3}{R_2 C_{eq}} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{C_{eq}} \left(\frac{C_2 + C_3}{R_1} + \frac{C_3}{R_2} \right) & \frac{1}{C_{eq}} \left(\frac{C_2 + C_3}{2R_1} - \frac{C_3}{2R_2} \right) \\ \frac{1}{C_{eq}} \left(\frac{C_1 + C_3}{R_2} + \frac{C_3}{R_1} \right) & \frac{1}{C_{eq}} \left(\frac{C_3}{2R_1} - \frac{C_1 + C_3}{2R_2} \right) \end{bmatrix}$$

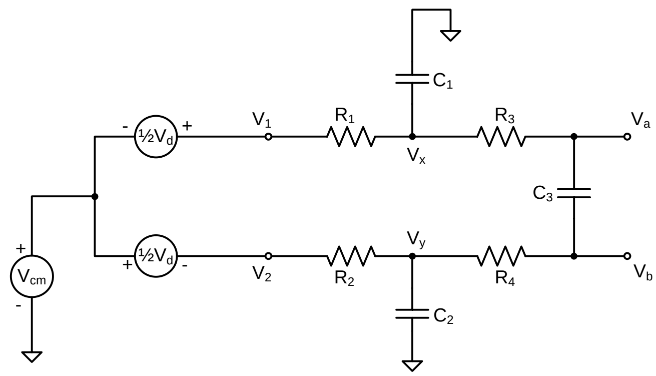
$$C = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where $C_{eq} = C_1 C_2 + C_1 C_3 + C_2 C_3$

Where the inputs, states, and outputs are:

$$\mathbf{u} = \begin{bmatrix} V_{cm,in} \\ V_{diff,in} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} V_a \\ V_b \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} V_{cm,out} \\ V_{diff,out} \end{bmatrix}$$

<u>Case 3:</u> Case 1 followed by a differential low-pass



KCL at node V_x: $\dfrac{V_1 - V_x}{R_1} = C_1 V_x' + \dfrac{V_x - V_a}{R_3}$ ➔ $V_x{}' = -\left(\dfrac{1}{R_1C_1} + \dfrac{1}{R_3C_1}\right)V_x + \dfrac{1}{R_1C_1}V_1 + \dfrac{1}{R_3C_1}V_a$

KCL at node V_y: $\dfrac{V_2 - V_y}{R_2} = C_2 V_y' + \dfrac{V_y - V_b}{R_4}$ ➔ $V_y{}' = -\left(\dfrac{1}{R_2C_2} + \dfrac{1}{R_4C_2}\right)V_y + \dfrac{1}{R_2C_2}V_2 + \dfrac{1}{R_4C_2}V_b$

KCL at node V_a: $\dfrac{V_x - V_a}{R_3} = C_3\left(V_a' - V_b'\right) = C_3 V_{ab}'$ ➔ $V_{ab}{}' = \dfrac{1}{R_3C_3}V_x - \dfrac{1}{R_3C_3}V_a$

KVL around diff. filter: $V_x - V_y = \left(R_3 + R_4\right)i + V_{ab}$

Substitute in: $V_1 = V_{cm} + \dfrac{1}{2}V_{diff}$ and $V_2 = V_{cm} - \dfrac{1}{2}V_{diff}$

Substitute in: $V_a = \dfrac{R_4}{R_3 + R_4}V_x + \dfrac{R_3}{R_3 + R_4}V_y + \dfrac{R_3}{R_3 + R_4}V_{ab}$ and $V_b = \dfrac{R_4}{R_3 + R_4}V_x + \dfrac{R_3}{R_3 + R_4}V_y - \dfrac{R_4}{R_3 + R_4}V_{ab}$

In state-space form, it becomes...
$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$

$$A = \begin{bmatrix} -\dfrac{1}{C_1}\left[\dfrac{1}{\left(R_3 + R_4\right)} + \dfrac{1}{R_1}\right] & \dfrac{1}{C_1\left(R_3 + R_4\right)} & \dfrac{1}{C_1\left(R_3 + R_4\right)} \\ \dfrac{1}{C_2\left(R_3 + R_4\right)} & -\dfrac{1}{C_2}\left[\dfrac{1}{\left(R_3 + R_4\right)} + \dfrac{1}{R_2}\right] & -\dfrac{1}{C_2\left(R_3 + R_4\right)} \\ \dfrac{1}{C_3\left(R_3 + R_4\right)} & -\dfrac{1}{C_3\left(R_3 + R_4\right)} & -\dfrac{1}{C_3\left(R_3 + R_4\right)} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{R_1C_1} & \dfrac{1}{2R_1C_1} \\ \dfrac{1}{R_2C_2} & -\dfrac{1}{2R_2C_2} \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \dfrac{R_4}{R_3 + R_4} & \dfrac{R_3}{R_3 + R_4} & \dfrac{R_3 - R_4}{2\left(R_3 + R_4\right)} \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where the inputs, states, and outputs are:

$$\mathbf{u} = \begin{bmatrix} V_{cm,in} \\ V_{diff,in} \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} V_x \\ V_y \\ V_a - V_b \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} V_{cm,out} \\ V_{diff,out} \end{bmatrix}$$

The TF from CM input to differential output has one (1) zero (located close to zero) and three (3) poles.