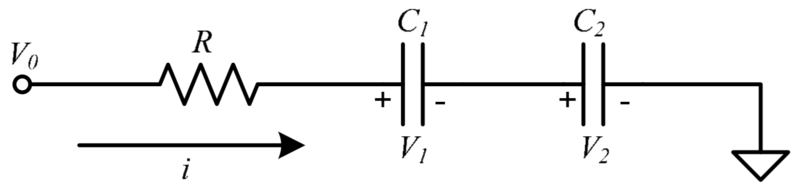

Summing voltages,

$$V_0 = Ri + V_1 + V_2 \tag{1}$$

The capacitor voltages are related to the current by,

$$i = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt} \tag{2}$$

Integrating this from time zero to time *t*:

$$C_1\left[V_1(t) - V_{1,0}\right] = C_2\left[V_2(t) - V_{2,0}\right] \tag{3}$$

Solving for $V_2(t)$:

$$V_2(t) = V_{2,0} + \frac{C_1}{C_2}\left[V_1(t) - V_{1,0}\right] \tag{4}$$

Plugging (2) and (4) into (1):

$$V_0 = RC_1\frac{dV_1}{dt} + V_1 + V_{2,0} + \frac{C_1}{C_2}\left[V_1 - V_{1,0}\right]$$

$$\Rightarrow \; RC_1\frac{dV_1}{dt} + \left(\frac{C_1 + C_2}{C_2}\right)V_1 = V_0 + \frac{C_1}{C_2}V_{1,0} - V_{2,0} \tag{5}$$

$$\Rightarrow \; \frac{dV_1}{dt} + \frac{1}{R}\left(\frac{C_1 + C_2}{C_1 C_2}\right)V_1 = \frac{1}{RC_1}\left(V_0 + \frac{C_1}{C_2}V_{1,0} - V_{2,0}\right)$$

The steady state solution is:

$$\begin{aligned} V_{1,ss} &= \frac{C_2}{C_1 + C_2}\left(V_0 + \frac{C_1}{C_2}V_{1,0} - V_{2,0}\right) \\ &= \frac{C_2}{C_1 + C_2}V_0 + \frac{C_1}{C_1 + C_2}V_{1,0} - \frac{C_2}{C_1 + C_2}V_{2,0} \end{aligned} \tag{6}$$

We can solve for $V_{2,ss}$ using (4):

$$
\begin{aligned}
V_2(t) &= V_{2,0} + \frac{C_1}{C_2}\left[\frac{C_2}{C_1+C_2}V_0 + \frac{C_1}{C_1+C_2}V_{1,0} - \frac{C_2}{C_1+C_2}V_{2,0} - V_{1,0}\right] \\
&= V_{2,0} + \frac{C_1}{C_2}\left[\frac{C_2}{C_1+C_2}V_0 - \frac{C_2}{C_1+C_2}V_{1,0} - \frac{C_2}{C_1+C_2}V_{2,0}\right] \\
&= V_{2,0} + \frac{C_1}{C_1+C_2}V_0 - \frac{C_1}{C_1+C_2}V_{1,0} - \frac{C_1}{C_1+C_2}V_{2,0} \\
&= \frac{C_1}{C_1+C_2}V_0 - \frac{C_1}{C_1+C_2}V_{1,0} + \frac{C_2}{C_1+C_2}V_{2,0}
\end{aligned}
\qquad (7)
$$

The general solution is:

$$V_1(t) = K_1 e^{-\frac{C_1+C_2}{RC_1C_2}t} + V_{1,ss} \qquad (8)$$

Applying the initial condition:

$$V_1(0) = V_{1,0} = K_1 + V_{1,ss} \;\Rightarrow\; K_1 = V_{1,0} - V_{1,ss} \qquad (9)$$

We can use (2) to solve for $i(t)$:

$$i(t) = C_1\frac{dV_1}{dt} = -C_1K_1\frac{C_1+C_2}{RC_1C_2}e^{-\frac{C_1+C_2}{RC_1C_2}t} = \left(V_{1,ss} - V_{1,0}\right)\frac{C_1+C_2}{RC_2}e^{-\frac{C_1+C_2}{RC_1C_2}t} \qquad (10)$$